Name ____

Partner(s) ____

worked in class

Activity 3: Relations and Functions

Watch the mini lecture in Canvas and complete the example problems before you begin the practice exercises. In this activity, we will learn about what relations and functions are and how to determine which relations are functions.

Relation: A set of ordered pairs (x, y) where each x is in the **domain** [set of inputs] and each y is in the **range** [set of outputs].

Function: A relation such that for each value of x in the **domain**, there is *exactly one value* of y in the **range**.

Example 1: Determine if the relation in the following example is a function.

Group X consists of the students in Math 106 and Group Y consists of the letters A, B, C, D, and F. The "grade" relation matches each student in Group A with a letter in Group B, according to the percentage earned by the student.

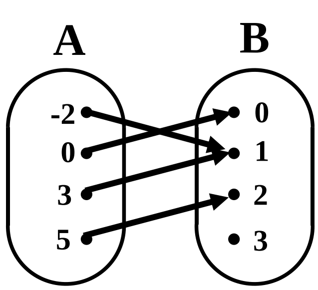
Function? YES or NO   Why or why not?

Example 2: Determine if each relation represents a function and state **why** you know.

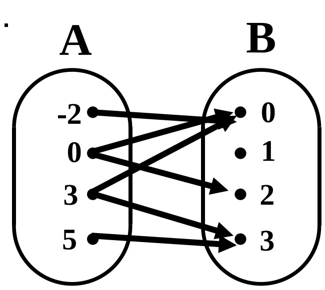
a. $\{(2,3),(-1,5),(3,5),(5,0)\}$

b. $\{(5,-1),(6,-3),(8,-5),(8,0)\}$

c.



d.

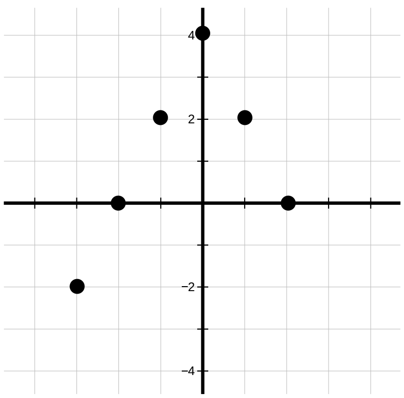


In example 2, we are shown relations in terms of ordered pairs. If you are instead given the graph of a relation on the Cartesian plane [xyplane], you can easily determine if the relation is a function using the Vertical Line Test:

Vertical Line Test: A graph represents a function if and only if it passes the **vertical line test**. Every vertical line intersects the graph at most _______ time(s).

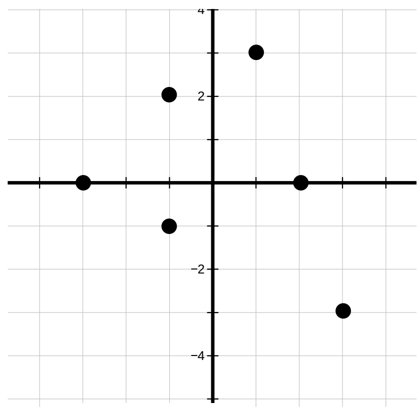
Example 3: Use the VLT to determine whether each relation below is a function. If the relation is not a function, draw a vertical line to show that the relation fails the VLT.

a.



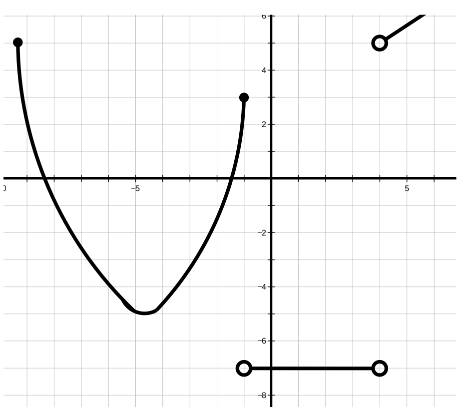
Function?

c.



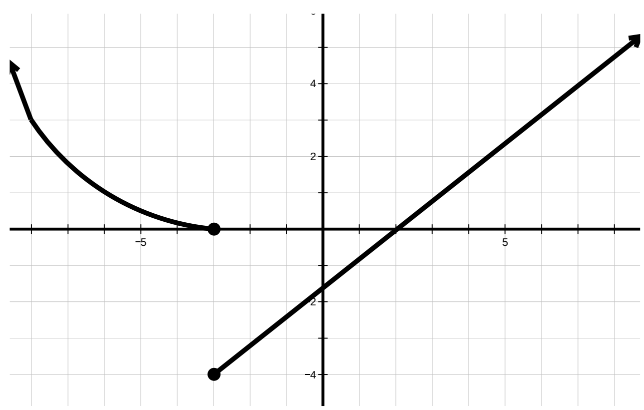
Function?

b.



Function?

d.



Function?

Example 4: Determine whether the following relations are functions by solving for y and then determining if any x-value will produce more than one y-value. Show all work.

a. $(x-3)^2 = y+4$

b. $(y-3)^2 = x+4$

c. $3x = |y| - 2$

d. $(y+1)^3 = 5x$

Function Notation is another way of representing y. For instance, we could write $y = 7x - 9$, or equivalently, $f(x) = 7x - 9$. You may also see functions written as $y = f(x) = 7x - 9$.

The nice thing about using function notation is that we can easily label values. For instance, if I want to know the value of y when $x = 2$, I can use the notation $f(2)$, which is read "*f* of 2."

**Note that function notation is different than multiplication. When we write $f(2)$, we mean:

$$f(2) = 7 \cdot 2 - 9 = 5$$

Function notation just shows us what ***substitution*** to make.

Example 5: Use the function $g(x) = 2x - 3$ to complete the following. Simplify if necessary.

a. $g(0) =$

b. $g(-1) =$

c. $g(5) =$

d. $g(y) =$

e. $g(a) =$

f. $g(m+2) =$

g. $g(x+h) =$

h. $g(\circlearrowleft) =$

i. $g(x) + g(h) =$

j. $g(x) + h =$

Practice Exercises

★ 1. Describe relation and function in your own words. Think about the similarities and differences in the definitions above to ensure you could identify a relation and a function using your definition. Compare your descriptions with at least two other students in class and adjust your descriptions, if needed.

Relation:

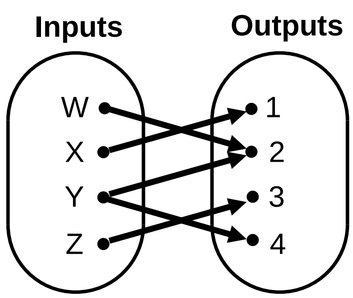
Function:

2. Determine whether the following relations are functions. Show all work and/or explain your reasoning.

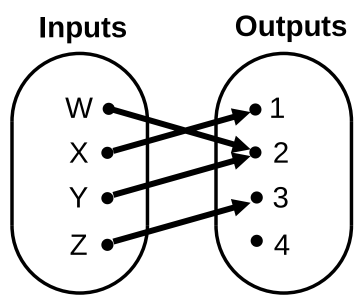
a. $\{(12, 4), (-5, 4), (4, -10), (-1, 7)\}$

b. $\{(6, 18), (-8, -2), (0, 11), (6, 3)\}$

c.



d.



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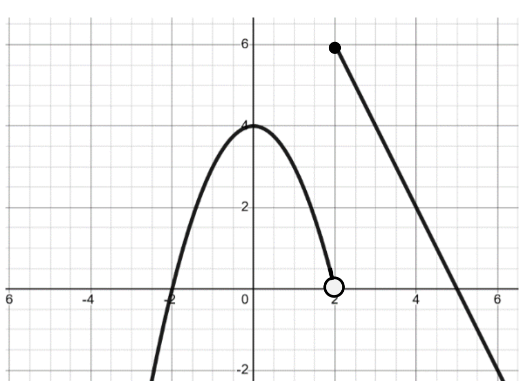
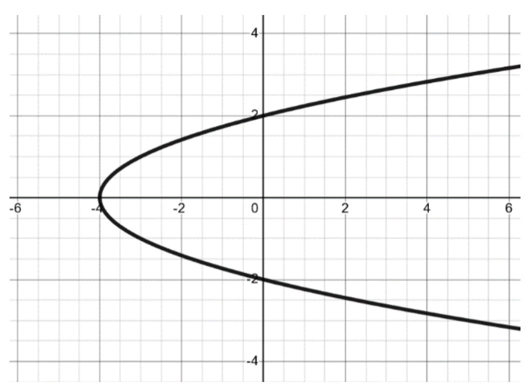
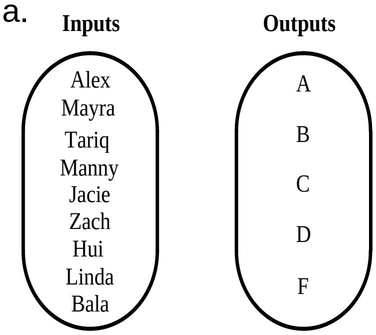


f.



★ 3. Why does the Vertical Line Test (VLT) work? In other words, explain how a vertical line intersecting more than one point on the graph shows that the relation is not a function.

4. Complete each diagram below so that there is a function relating the group on the left (inputs) to the group on the right (outputs). If it is not possible, state why.

a.


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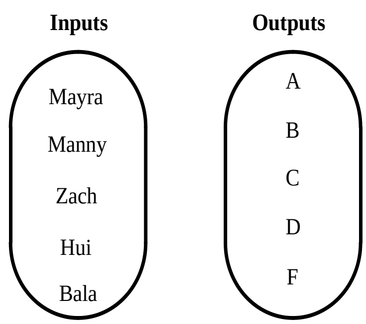
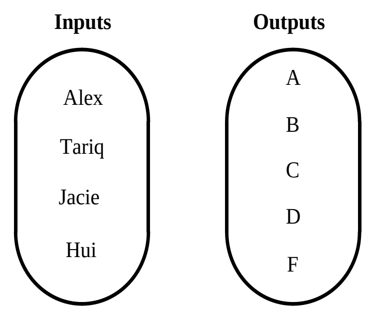


c.


5. Concisely describe what "y is a function of x" means in your own words. **Hint**: Think about Example 1 – why we would say that the letter/grade is a function of the student, but the student is NOT a function of the letter/grade. Compare your description with at least two other students in class and adjust your description, if needed.

6. Determine whether the following relations are functions by solving for y and then determining if any x-value will produce more than one y-value. Show all work.

   a. $(x-1)^2 = y^2$

   c. $7x - 2 = y^4$

   b. $2y + 3x = x - 5y + 4$

   d. $y^3 = 6x + 8$

7. Use the function $k(x) = x^2 + 1$ to complete the following. Simplify if necessary.

   a. $k(0) =$

   f. $k(m+2) =$

   b. $k(-1) =$

   g. $k(x+h) =$

   c. $k(5) =$

   h. $k(\Delta) =$

   d. $k(y) =$

   i. $k(x) + k(h) =$

   e. $k(a) =$

   j. $k(x) + h =$

★ 8. Use the graph of $f(x)$ to determine the following. Estimate the values if necessary.

a. $f(-2) =$

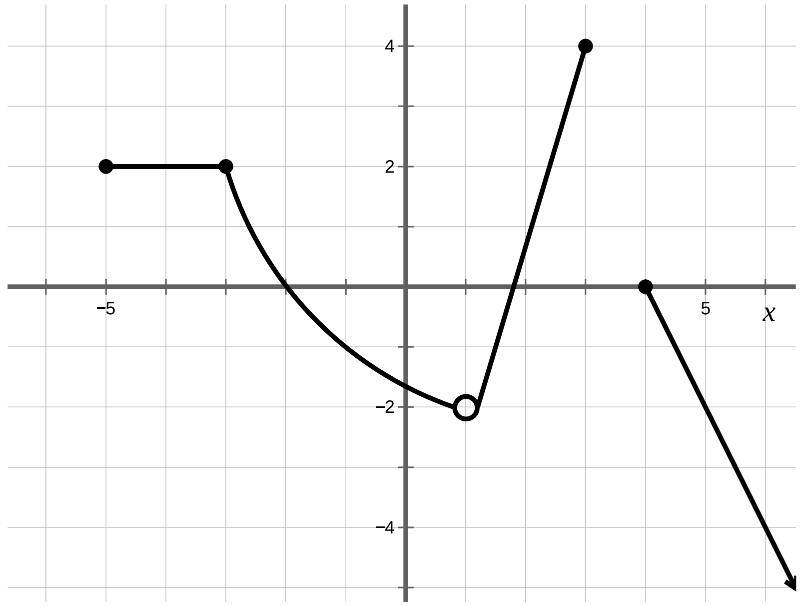
b. $f(3) =$

c. $f(-4) =$

d. $f(1) =$

e. if $f(x) = -1, x =$

f. if $f(x) = -2, x =$



g. Given any function $y = f(x)$, the $\boldsymbol{x}$ **–intercepts** are determined by the real solutions to the equation $f(x) = 0$. Name the part of the graph where you find the x –intercepts of $f(x)$.

h. List any x –intercepts of $f(x)$ shown on the graph above. Estimate the values if needed. Each x –intercept should be written in the form $(x, 0)$.

i. Given a function $y = f(x)$, the $\boldsymbol{y}$ **–intercept** is determined by $f(0)$. Name the part of the graph where you find the y –intercept of $f(x)$.

j. List the y –intercept of $f(x)$ on the graph above. The y –intercept should be written in the form $(0, y)$.

k. Why can there only be one y –intercept for a function?

9. For each function below, find any $x-$ and $y-$intercepts and list them. If there are none, write NONE. Show all work. Hint: Revisit the previous problem to determine how to find the intercepts.

a. $g(x) = x^2 - 5x - 6$

$x-$intercept(s): ___________________

$y-$intercept: _____________________

b. $f(x) = x^2 - 6x + 9$

$x-$intercept(s): ___________________

$y-$intercept: _____________________

c. $j(x) = \sqrt{6 - x}$

$x-$intercept(s): ___________________

$y-$intercept: _____________________

d. $k(x) = \frac{3x+8}{x+5}$

$x-$intercept(s): ___________________

$y-$intercept: _____________________